Stats 2 - June 2013

① a) $\bar{x} = \frac{8532}{9} = 948$

$s^2 = \frac{38538}{8} = 4817.25$

$\nu = n-1 = 9-1 = 8$

t value$_{(8)}$, 2 tailed, 98% $= 2.896$

Confidence Interval $= 948 \pm 2.896 \times \sqrt{\frac{4817.25}{9}}$

$= 948 \pm 67.00$

$= (881.0, 1015.0)$

b) i) μ = midpoint of interval $= (927 + 1063) \times \frac{1}{2} = 995$

ii) Because the two confidence intervals overlap, we do not have enough evidence to suggest mean weights have changed

② a) Expected

	Apply	Not Apply
Letter	35.2	124.8
Phone	8.8	31.2

Yates' Correction

$(|O - E| - 0.5)^2 / E$

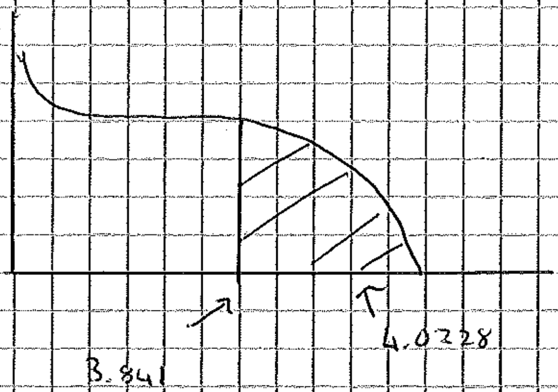
	Apply	Not Apply
Letter	0.6276	0.1770
Phone	2.5102	0.1080

H_0: No association between how they communicated and the outcome

H_1: There is an association

Test Statistic: $\Sigma X^2 = 4.0228$

Critical Value: X^2, $\nu = 1$, 5% $= 3.841$



$4.0228 > 3.841$

$\therefore$ Reject H_0

Evidence suggests that phone calls' effects were higher than expected, which supports the council's beliefs.

b) Type I error
Ho has been rejected when it was true.

(3) a) i) Quickest she can get there:

$2 + 0 + 20 + 5 = 27$

ii) Waits 10 mins per tram $\rightarrow$ 37

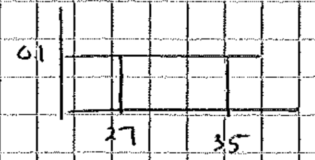
b) Rectangular Distribution:

$E(T) = \frac{1}{2}(27 + 37) = 32$

$Var(T) = \frac{1}{12}(37 - 27)^2 = \frac{100}{12} = 8\frac{1}{3}$

c) $P(25 \leq T \leq 35) = P(27 \leq T \leq 35)$

$= 8 \times 0.1 = 0.8$



(4) a) i) $P(G=4) \rightarrow \frac{e^{-3.5} \times 3.5^4}{4!} = 0.189$

ii) Each day $\rightarrow$ $GD \sim Po(0.5)$ $\quad 3.5 \div 7$

$P(GD \geq 2) = 1 - P(GD \leq 1)$

$= 1 - 0.9098 = 0.0902$

iii) 28 days $\rightarrow$ $G \sim Po(14)$ $\quad 28 \times 0.5$

$P(10 < G < 20) = P(G \leq 19) - P(G \leq 10)$

$= 0.9235 - 0.1757 = 0.7478$

b) Explosions should be random.

(5) a) i)

x	1	2	3	4	5
$P(X=x)$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	k

$k = 1 - \left(\frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}\right) = \frac{1}{20}$

ii) $E(X) = 1 \times \frac{1}{3} + 2 \times \frac{1}{4} + 3 \times \frac{1}{5} + 4 \times \frac{1}{6} + 5 \times \frac{1}{20}$

$= 2.35$

$E(X^2) = 1^2 \times \frac{1}{3} + 2^2 \times \frac{1}{4} + 3^2 \times \frac{1}{5} + 4^2 \times \frac{1}{6} + 5^2 \times \frac{1}{20}$

$= 7.05$

$Var(X) = 7.05 - (2.35)^2$

$= 1.5275$

iv) $P(X > 2) = 1/5 + 1/6 + 1/20 = 5/12$

b) $E(Y) = 100 \times E(X) - 50$

$= 100 \times 2.35 - 50 = 185$

$sd(Y) = 100 \times sd(X) = 100 \times \sqrt{1.5275}$

$= 5\sqrt{611} = 123.59..$

(6) a) $H_0: \mu = 175$

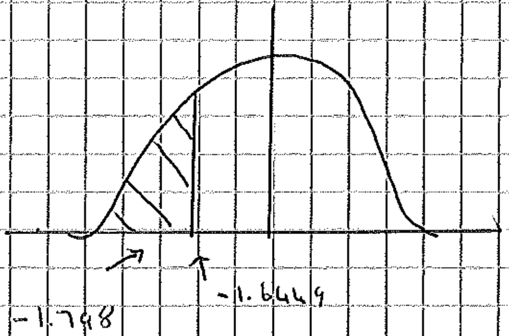
$H_1: \mu < 175$ (1 tailed test)

$\bar{x} = 168.1$

$\sigma = 9.4$ ← we know σ, so must use Z

Test statistic $= \dfrac{168.1 - 175}{9.4/\sqrt{6}} = -1.798$

Critical value: Z, 5%, 1 tailed test $= -1.6449$



$-1.798 < -1.6449$

$\therefore$ Reject H_0

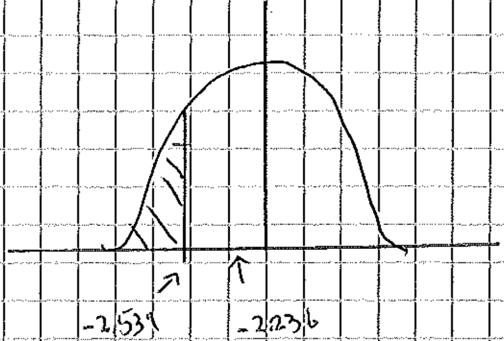
There is evidence at 5% level that mean weight of the pears has dropped.

b) $\bar{x} = 169.4$ $\quad n = 20$ $\quad H_0: \mu = 175$, $H_1: \mu < 175$

$s = 11.2$ $\quad v = 19$ ← must use t as don't know σ

Test statistic $= \dfrac{169.4 - 175}{11.2/\sqrt{20}} = -2.236$

Critical value, t, (19), 1%, 1 tailed test $= -2.539$



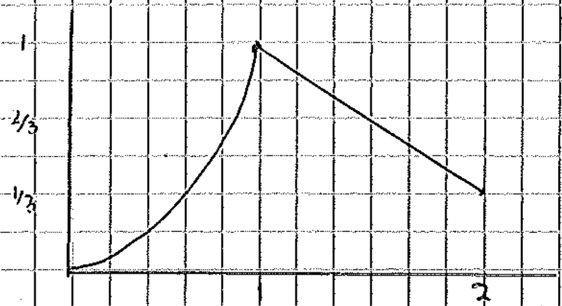
$-2.236 > -2.539$

$\therefore$ Accept H_0

Not enough evidence at 1% level to support claim that the mean weight has fallen.

c) Because significance level is smaller.

7) a)



b) i) $F(x) = \int_0^x f(x) = \int_0^x x^2 = x^3/3$ or $1/3\, x^3$

ii) $F(x) = 0.25 \rightarrow 1/3\, x^3 = 0.25$

$\rightarrow x^3 = 0.75 \rightarrow x = 0.9085$...

c) i) $1/3 \int_1^x 5 - 2x = 1/3 \left[5x - x^2\right]_1^x$

$= 1/3 \left[(5x - x^2) - (5 - 1)\right]$

$= 1/3 \left[5x - x^2 - 4\right]$

Need to add on $F(1) = 1/3(1)^3 = 1/3$

$\rightarrow 1/3\left[5x - x^2 - 4\right] + 1/3$

$= 1/3\left[5x - x^2 - 3\right]$

ii) $F(x) = 0.75 \rightarrow 1/3\left[5x - x^2 - 3\right] = 0.75$

$\rightarrow 5x - x^2 - 3 = 2.25$

$\rightarrow x^2 - 5x + 5.25 = 0$

$\rightarrow 4x^2 - 20x + 21 = 0$

$(2x - 3)(2x - 7) = 0$

$x = 3/2$ $\qquad$ $x = 7/2$

UQ must be $3/2$ as $7/2$ is out of range